tory, Cape of Good Hope; G. E. Hale, The Study of Stellar Evolution, presented by the author; five spectroheliographs (enlargements), showing hydrogen and calcium flocculi, presented by the Mount Wilson Observatory; lithograph from a drawing of Donati's Comet, by Miss Charlotte S. Cooper, Markree, presented by Miss Cooper; 16 charts of the Astrographic Chart of the heavens, presented by the Royal Observatory, Greenwich.

Considerations on the Form and Arrangement of New Tables of the Moon. By Simon Newcomb.

Hansen's Tables of the Moon, with some patching up, have now been in use in the standard astronomical ephemerides for a full half century. With every passing year the necessity of replacing them by improved ones becomes more urgent. My own investigations on the Moon's motion have always been carried on with the view of ultimately constructing new tables, while experience in the use of Hansen's tables has from time to time formed a basis for a study of the relative merits of various forms of the lunar theory for the purpose of tabulation. But, as the years have passed, it has become increasingly apparent that I must leave to other hands the execution of the desired work. I therefore venture to summarise the suggestions which I have to make on the whole subject.

§ 1. *General Form of Tables.*

All my experience with Hansen's tables has led me to the conclusion that if the problem were only that of enabling an ephemeris of the Moon to be computed with the use of the fewest figures, it would scarcely be possible to improve on Hansen's arguments and system of tabulation. The Hansenian perturbations of the mean anomaly are more rapidly convergent than those of any co-ordinate that can be used, and therefore require fewest tables. The expression of the fundamental argument in terms of the time may also have a certain advantage, though this is a point on which I feel less confident, because the best unit is not a decimal of a day.

But the mental labour to be performed is not measured merely by the number of figures taken out and written down. A "straight-ahead" computation has a decided advantage over one in which it is necessary to form numbers from preceding tables, or to introduce tables of logarithms, as we must when we pass from Hansen's fundamental argument to the true longitude. This remark is especially applicable to the reduction to the ecliptic. The mental labour of forming arguments from numbers already used and of combining results from various tables is intensified by the care and attention which then have to be bestowed, and which it is desirable to render unnecessary.

The desired uniformity in proceeding would be attained in the highest degree by tabulating the perturbations of the ecliptic longitude and latitude in the form to which they are reduced by Brown, using a uniform system of arguments for the tables. But the number of tables then requisite would be so great that it is doubtful whether they could be put into a single volume of reasonable size, say that of Hansen's tables as they now are. The most troublesome and important terms would be those arising from the reduction to the ecliptic. It may be added that I have never found Hansen's system of constructing the double-entry tables for intervals of $0^d\cdot25$ to offer a great advantage over the use of 12-hour intervals, except when an isolated place is to be computed.

The modification of the plan which I have to suggest will best be seen by starting with Hansen's fundamental argument. After it is formed, two steps are required—the reduction from the inequality of mean longitude to that of true longitude in orbit, and the reduction to the ecliptic. When the reductions are made in the general formulæ, tabulated, as they will be if ecliptic co-ordinates are tabulated, each of these steps adds a number of additional inequalities, but I think those arising from the second reduction exceed in number and magnitude those arising from the first. However this may be, it is certain that the tabulation of the true longitude will require much more voluminous tables than will that of the Hansenian mean longitude.

What I would suggest is, therefore, the tabulation of the longitude in orbit, that co-ordinate being defined as the distance from the mean node to the circle of latitude passing through the Moon perpendicular to the ecliptic. More geometric symmetry would be attained by taking as the fundamental plane the mean moving orbit of the Moon, and referring the longitude and latitude to that plane. But this would require a multiplication of the quantities by certain factors in order to find the true longitude. If the principal term of the latitude is computed with the true argument of latitude, the inequalities are fewer and smaller than if the mean argument is used.

§ 2. *Unit of Tabular Longitude.*

In the case of the longitude, the next question is that of the unit to be chosen. It is so important to avoid the use of larger numbers than is necessary that this choice requires careful consideration. The tabulated unit of Hansen's fundamental argument is approximately $0''\cdot005$. The smallness of this quantity makes the interpolation laborious without really adding materially to the accuracy of the result. To devise the best unit, we must first consider the degree of precision to be reached by some arbitrarily chosen unit, say $0''\cdot01$. Here some statements relating to the accumulation of small accidental errors become necessary.

§ 3. *Errors accumulating from omitted decimals.*

The theory of errors accumulating from omitted decimals is not usually developed in a way that seems to me entirely satisfactory. I therefore begin by stating some principles, definitions, and results pertaining to the subject.

1. I use the term *probable value* of a doubtful quantity in the sense introduced by Poincaré, which is—

If
$$v_1, v_2, \ldots . v_n$$

be all the essentially different possible values of a quantity v, and if

$$p_1, p_2, \ldots . p_n$$

be the several probabilities of these values, then the probable value of v is that given by the equation

$$v = p_1 v_1 + p_2 v_2 + \ldots . + p_n v_n .$$

That is, the probable value is the mean of all the possible values, each weighted according to its probability.

2. I deal, in the case of each quantity, with the probable value of the square of its deviation from the truth. The square root of this square is generally designated in astronomy as the *mean error*. Pearson has introduced the somewhat more appropriate term of "standard deviation," which I abbreviate to S.D. We may therefore define this as the square root of the probable value of the square of the deviation.

3. Let us consider the sum S of any number n of independent quantities, $q_1, q_2, q_3, \ldots . q_n$.

Let these q's be uncertain by the several standard deviations

$$\epsilon_1, \epsilon_2, \epsilon_3 \ldots . \epsilon_n .$$

If we put ϵ for the S.D. of S we shall then have

$$\epsilon^2 = \epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2 + \ldots . + \epsilon_n^2 .$$

The advantage of this equation is that it is independent of the law of probability of an error as a function of its magnitude, which is not in all cases the normal law. For example, the error arising from the omission of a periodic term has a well-defined absolute maximum equal to the coefficient of the term. Moreover, its maximum value is more probable than any other. The standard deviation arising from the omission of a periodic term $a \sin mt$ is

$$\epsilon = \pm \frac{a}{\sqrt{2}} .$$

An examination of the coefficients of longitude found by Brown and Hill shows that there are about 300 periodic terms whose

coefficients lie between the limits $0''\cdot01$ and $0''\cdot001$. I take this lower limit as that beyond which it is unnecessary to go. In estimating the S.D. I take $0''\cdot01$ as the unit. An approximate estimate of the distribution of these coefficients in magnitude shows that the square of the S.D. produced by omitting them is, with sufficient exactness,

$$\epsilon^2 = 28\,.$$

There remain, by a rough estimate, about 300 terms exceeding the limit $0''\cdot01$, nearly all of which I assume will be tabulated. I also assume that the individual numbers of these tables will be formed by carrying each number to $0''\cdot001$, and that the last decimal will be dropped in the final tabulated number, which will be given only to $0''\cdot01$. Making due allowance for all imperfections, I find that the standard deviation of a number interpolated from a single table thus formed will be $\pm 0\cdot26$ if the number is written down to one decimal beyond that of the tables. But if the number is, as usual, only written to the tabular number of decimals, the deviation will be $\pm 0\cdot39$.

I assume that not more than 120 tabular numbers will be added to form the longitude. The contribution to $(\text{S.D.})^2$ arising from the summation of these numbers is

$$120 \times 0\cdot39^2 = 18\cdot2\,.$$

The imperfections of the arguments will also have their influence. The deviation produced by them may be reduced by giving each argument to one decimal beyond that required by the condition that the error arising from a unit error of the argument shall always be less than that of the tabular unit. But, without going beyond this rule, the effect of the errors of the argument will not exceed that arising from the addition of thirty more tabular numbers. We may thus have—

$$(\text{S.D.})^2 \text{ from errors of arguments } = 4\cdot5.$$

In some cases the effect of the imperfections of the correction for the second differences may add to the S.D. I think that, with a little skill and attention on the part of the computer, this S.D. need not exceed $\pm 0\cdot2$, giving $4\cdot8$ for its entire contribution to $(\text{S.D.})^2$.

Summing up all the sources of accidental deviation of the tabular results from theory we have—

From the omission of small terms . .	$(\text{S.D.})^2 =$	28
,, ,, tabular decimals .	,,	18
,, errors of arguments . . .	,,	5
,, ,, of second diff. . .	,,	5
In all	$(\text{S.D.})^2 =$	56

And for the sum, S.D. $= 7\cdot5$.

We shall therefore have, in the case of each computed longitude, a S.D. of $\pm 0''\cdot075$ and a probable error of $0''\cdot05$.

§ 4. *Degree of precision required.*

Let us compare this with the degree of precision required in a comparison with observation. My experience in the use and examination of lunar observations leads me to the conclusion that no single observation of any sort can be practically made without a mean error exceeding $\pm 0''\cdot5$, corresponding to a probable error of $\pm 0''\cdot34$. It is desirable that the mean error of a co-ordinate found from the tables should be less than this. But all the results obtained from lunar observations depend upon a great number of observations, which make the accidental errors unimportant in comparison with the systematic ones. The practical advantage of a degree of precision above that just mentioned in the tabular places is very small, and is practically evanescent if reduced below $\pm 0''\cdot4$. This degree of precision will be surpassed by adopting as the unit to be tabulated the 10^{-8} part of the circumference, or $0''\cdot01296$. Of course the unit $0''\cdot01$ would answer our purpose if deemed more convenient in use. But the smaller the numbers used, the easier the computer will find it to avoid small errors, while the S.D. will still fall below $\pm 0''\cdot09$.

There is, however, one point still to be considered in this connection which may modify our conclusions. Granting an S.D. of $\pm 0''\cdot10$ in the individual longitudes, we must expect that in the course of a year there may be three or four of the 730 tabulated longitudes in error by three times the S.D., and possibly one of four times this amount. But every error approaching such a magnitude as this will be detected by differencing the 12-hour ephemeris. A legitimate proceeding will then be to smooth off the ephemeris by such small corrections as shall reduce the higher, say the fifth or sixth, differences to a sufficiently smooth series. Each corrected tabular result may then be regarded as the mean of two or more neighbouring quantities, and the maximum error of the ephemeris will be reduced nearly to the mean S.D. In a word, we may fairly count on having an ephemeris in which all the errors exceeding some limit between $0''\cdot12$ and $0''\cdot30$ will be eliminated. This limit is still within the errors of the best observations, and the cases in which it is approached will be rare.

§ 5. *Reduction to sexagesimal units.*

The proposed units will require the reduction of the final longitudes to degrees, minutes, and seconds. The tables necessary for this purpose will perhaps fill four pages, and the computation will be equivalent to the entry of three additional tables.

If the unit $0''\cdot01$ is deemed preferable, its use will still require some study. It was adopted in Peirce's Tables of the Moon,

published in 1853. The tabular numbers were there expressed in degrees and seconds, minutes being ignored. I found the use of this system cumbrous, and should prefer to use seconds pure and simple, subtracting 1,296,000″ or its multiples when necessary. The proposed circumferential unit does away with this subtraction. Although it is a little easier to change the degrees into seconds and minutes than it is to change the circumferential unit, I still think the advantage to lie with the latter.

It may be of interest in this connection to note that if we should base the unit on the degree, tabulating to 0°·00001, the S.D. of the individual longitudes would still be only ±0″·18, and we might be fairly confident that no error exceeding 0″·4 would remain in a smoothed-off annual ephemeris as often as once a year.

Of course all this presupposes that the computer is always careful never to make a greater error than 0·5 in interpolating and writing down his number. The question may arise whether it is not well to allow him a margin of one or two units, by adopting smaller units. My answer is that the labour of handling large numbers involves more mental strain than that required in the accurate handling of small numbers, and that the assigned standard of precision will be more easily reached by the careful use of the smaller numbers than by the careless use of the larger ones.

§ 6. *Epochs and Arguments.*

For the practical work of computing places of the Moon for given dates, I do not think that any system more convenient than the usual one can be devised. The Hansenian form, in which the Gregorian and Julian calendars are used, is the most convenient of all. But it is always desirable to give the tables such a form that the relation between the tabular numbers and the original elements shall be easily examined, and corrections to the theory readily applied. This suggests a slight sacrifice of ease in computing an isolated position, or an ephemeris, to the requirements of the theoretical investigator.

Simplicity in the other direction is reached by the use of days of the Julian period. This was first employed, I believe, by Peirce, and is now extensively used in astronomy, especially in Oppolzer's works relating to eclipses. In using this system, a first and easy step is the reduction of the ordinary calendar date to days of the Julian period. Then absolute uniformity is reached in the construction and use of the tables.

The principal immediate drawback of this system is that, if used unmodified, the period of 1000 days must take the place of the year. The formation of the arguments of short period is then inconvenient. Many of the lunar arguments have periods not differing much from a month. From 12 to 15 multiples then suffice when the year is used, but with the period of 1000 days the number of multiples to be tabulated and subtracted will frequently be between 30 and 40, and some times more. Of course

this difficulty can be lessened by taking 500 days instead of 1000 as the second unit. But this will detract from simplicity of form.

A yet more serious drawback to the theoretical investigator is that the fundamental epochs usually adopted in astronomy, and for which the elements must be found, do not correspond to any power of 10 in the days of the Julian period. A complete transformation of the elements is therefore required to form the numbers on which the tables are based. If, therefore, multiples of 500 or 1000 days are used instead of years, I should prefer to count them back from 1900·0, thus gaining all the advantages of the Julian period without any other disadvantage than that of non-correspondence with the eclipse and other tables of Oppolzer.

The reduction of such a system to the ordinary calendar may be made a very simple matter. It seems to me, therefore, that the maximum of advantage will be reached by giving the fundamental arguments for cycles and periods based on multiples of 500 days before and after the fundamental epoch 1900 Jan. 0.

Probably the most convenient fundamental quantities to tabulate will be the longitude of the node, and the mean distances of the Moon and of its perigee from the node, all expressed in circumferential units. Then, whatever form the tables may be thrown into, we shall have the nearest approach to a simple, straight-ahead computation.

Finally, a serious problem is that of summing perhaps 100 periodic terms with coefficients not differing greatly from $0''{\cdot}01$. I have devised a machine for this purpose, the description of which must form the subject of another publication.

An Example of Professor Karl Pearson's Calculation of Correlation in the case of the Periodic Inequalities of Long-period Variables. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. The following note is written with a twofold purpose. Firstly, it is hoped that an indication of some value has been obtained with regard to the features of "long-period" variability; and secondly, the opportunity is taken to write out in full a simple example of the calculation of "correlation" between quantities by the methods of Professor Karl Pearson.

In the *M.N.* for March last (p. 416) Professor Pearson himself gave an admirable summary of methods; but he naturally did not repeat the elementary working which has become so familiar to him, and has been given often before in other connections. There are doubtless many to whom this working is already familiar; but there are certainly many others who do not know it and who might use it if they had an astronomical example readily accessible. In these busy days many people have not the leisure to search for references in scientific literature outside their own subject.